Spontaneous stochasticity in the Armstrong-Vicol passive scalar

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Résumé

Predictability in deterministic systems is often limited by chaos: nearby trajectories diverge exponentially in time, and the largest Lyapunov exponent sets a predictability horizon. Yet chaos preserves determinism-if initial conditions were known with infinite precision, future states would remain predictable, apparent randomness only reflects incomplete information. Some multiscale transport problems defy this paradigm and display finite-time loss of predictability, opening the door to a behavior known as spontaneous stochasticity. A classical signature of spontaneous stochasticity is Richardson pair dispersion: relative separation of passive tracers in a turbulent flow grows algebraically in time over an inertial range of scales, so that the time to reach macroscopic separation becomes essentially independent of the initial separation. This is the Lagrangian form of spontaneous stochasticity. A fluctuation–dissipation relation (T.D. Drivas, G.L. Eyink, JFM, (2017)) links Lagrangian spontaneous stochasticity to anomalous diffusion of the passive scalar: scalar-variance dissipation remains finite as the molecular diffusivity vanishes. Armstrong and Vicol (S. Armstrong, V. Vicol, Annals of PDE, (2025)) constructed explicitely a deterministic, divergence-free multiscale velocity field for which they can rigorously prove anomalous diffusion of the passive scalar. Their analysis also proves taht the vanishing diffusivity limit is a not selection principle. Depending on how the inviscid limit is taken, the passive scalar converges to different weak solutions of the inviscid transport equation. This opens the door to Eulerian Spontaneous stochasticity, i.e. randomness at the level of the scalar field itself. In (W. Ruffenach, E. Simonnet, N. Valade, Arxiv preprint, (2025)), we study both numerically and mathematically the construction of Armstrong and Vicol. First, we provide a numerical construction of the intricate Armstrong-Vicol vector field $\mathbf{b}$ and document anomalous diffusion together with Richardson-type dispersion and non-Dirac backward transition probabilities. We also prove rigorously that the advection-diffusion dynamics selects a non-Dirac probability measure on inviscid scalar solutions, establishing Eulerian spontaneous stochasticity of the passive scalar field.

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