Where Waves Concentrate: A Statistical Mechanics Theory for Inhomogeneous Moving Media

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Résumé

We present a statistical mechanics approach to predict the spatial statistics of short waves propagating in inhomogeneous moving media. By adapting the microcanonical framework of equilibrium statistical mechanics, we derive the local wave spectrum at any location in the domain from an ergodic prescription for the action density in phase space, constrained by conservation of absolute frequency. The approach applies to steady background flows and inhomogeneities, and requires no adjustable parameters. From the local wave spectrum, one can directly infer the spatial distribution of a wide range of wave statistics, including rms surface elevation, rms slope, kinetic energy, and Stokes drift in the case of hydrodynamic waves. We illustrate the method for shallow-water waves over currents and topographic inhomogeneities, as well as for deep-water surface capillary waves in the presence of a background flow. Predicted maps of rms surface elevation and interfacial slope show excellent agreement with numerical simulations.

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