Periodic solution computation in conservative systems: Application to the Mckean-Vlasov model.

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Résumé

In this work, we investigate the application of classical Newton and Newton–Picard methods to the computation of periodic solutions of temporal or spatio-temporal systems.
For conservative systems-those possessing one or more first integrals-such as the McKean–Vlasov model, the standard formulations of these methods do not inherently preserve the relevant invariants, and periodic orbits typically form families parameterized by conserved quantities. Building on the continuation framework for periodic orbits in conservative dynamical systems developed in (1), we apply these ideas to incorporate conserved quantities directly into Newton- and Newton–Picard-type iterations, treating linear invariants (e.g., mass) explicitly.

The resulting mass-preserving algorithms are demonstrated on the McKean–Vlasov model, which, as the overdamped limit of the Vlasov–Fokker–Planck equation (2), provides a fundamental mean-field setting for developing and validating invariant-preserving continuation methods relevant to electron bunch dynamics in particle accelerators(3).

References:

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