Purely Dilatational Burgers Turbulence: The Infinite-Mach-Number Limit of Compressible Turbulence

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Résumé

Understanding how energy transfers across scales in compressible turbulence remains a major open problem. The difficulty arises not only from the coupling between velocity and thermodynamic variables, but also from the presence of an additional dilatational mode in the velocity field, due to the loss of the divergence-free constraint. While the energy transfer of the solenoidal(incompressible) mode is well described by the Kolmogorov cascade picture, the dynamics of the dilatational mode remain poorly understood and call for further investigation.
As a simplified framework, purely potential Burgers turbulence provides a valuable mathematical model for isolating and studying dilatational dynamics. This system can also be viewed as the infinite-Mach-number limit of the weakly compressible turbulence model proposed by Marion et al. (1). In our study, we investigate the statistical properties, conservation laws, and energy transfer mechanisms of purely dilatational Burgers turbulence.

We first demonstrate that, in contrast to the solenoidal mode, which will generate dilatational mode naturally, the dilatational mode in Burgers turbulence behaves as an equilibrium mode. We further show that while one-dimensional Burgers turbulence conserves kinetic energy, energy conservation is broken in two- and three-dimensional cases. However, we identify an alternative inviscid conserved quantity, namely the spatial integral of the product of kinetic energy and velocity divergence.

To characterise scale-to-scale transfers, we then apply an Eddy-Damped Quasi-Normal Markovian (EDQNM) closure to the purely dilatational Burgers system. This approach allows us to analyse the spectral energy budget and investigate the resulting scaling laws. This provides new insights into the role of dilatational modes in turbulent energy transfer.

(1)Marion J.-D., Bertoglio J.-R, Mathieu J., Spectral study of weakly compressible turbulence: Part I, Direct Interaction Approximation, C. R. Acad. Sci. Paris, sdrie II 307 (1988) 1487-1492.

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